About GUIs for proofs in geometry.

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GUIs for interactive theorem proving

- Geometry Tutor
- Mentoniezh
- Chypre
- Cabri-Euclide
- Geometrix
- Baghera

GUIs for automated theorem proving

- MMP-Geometer
- Geometry Expert
- Geometry Explorer
- GEOTHER

Teaching community

- Prolog based expert systems
- Implicit theorems, hidden theorems,. . .

ATP community

- Ad-hoc theorem provers

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- There are facts than can not be visualized graphically and there are facts that are difficult to understand without being visualized.
- We should have both the ability to make arbitrarily complex proofs and use a base of known lemmas.
- The verification of the proofs by the proof assistant provides a very high level of confidence.

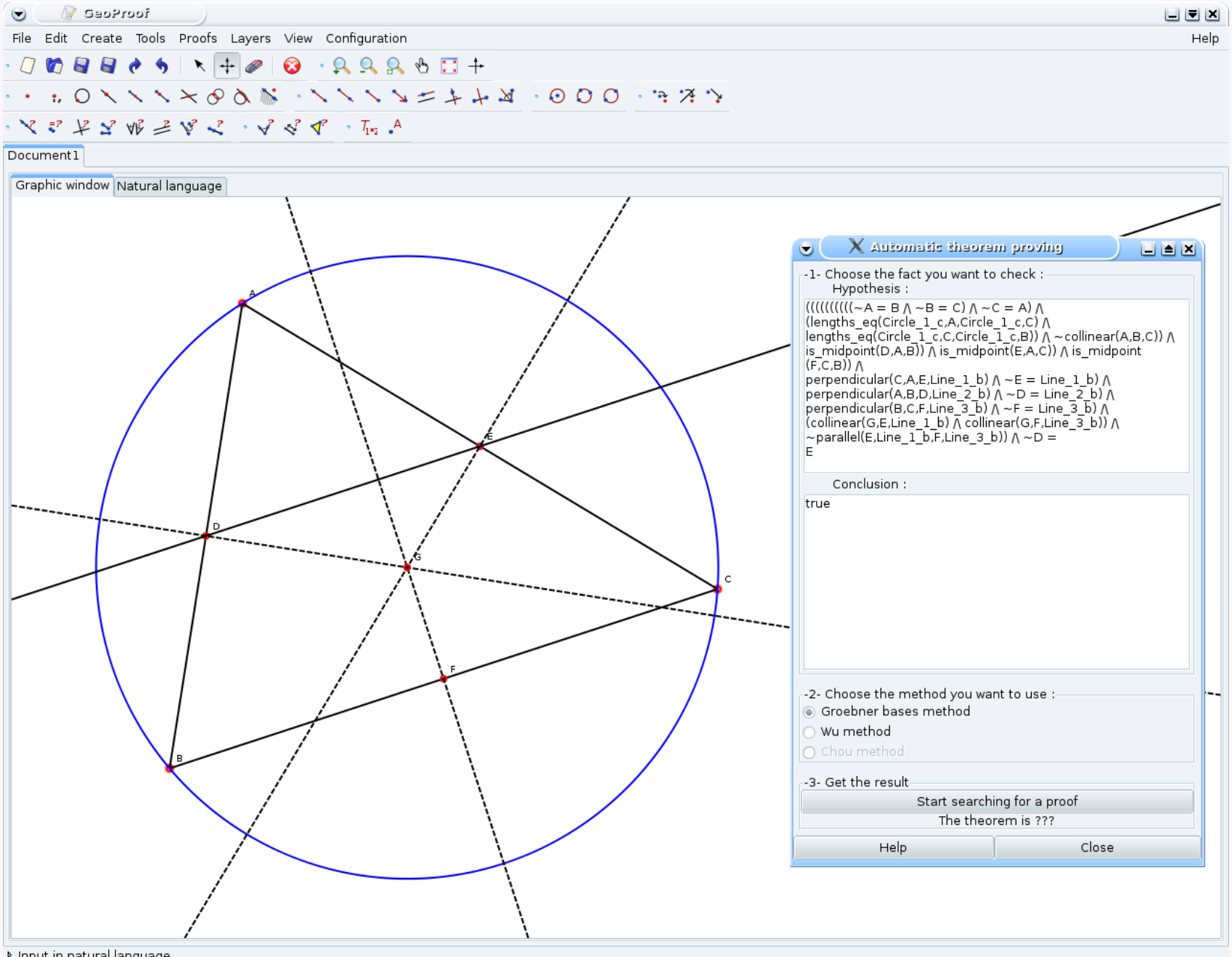
GeoProof combines these features:

- dynamic geometry
- automatic theorem proving
- interactive theorem proving (using Coq/CoqIDE)

Overview of GeoProof

- Ocaml, LablGTK2, Cairo
- License: GPL2
- Multi-platform (Linux, MacOSX, Windows)

Overview of GeoProof



Dynamic geometry features

- points, lines, circles, vectors, segments, intersections, perpendicular lines, perpendicular bisectors,angle bisectors...
- central symmetry, translation and axial symmetry
- traces
- text labels with dynamic parts:
  - measures of angles, distances and areas
  - properties tests (collinearity,orthogonality,...)

- **layers**
- Computations use **arbitrary precision**
- Input: XML
- Output: XML, natural language, SVG, PDF, PS, PNG, BMP, Eukleides (**latex**), **Coq**

Missing features:

- loci and conics
- macros
- animations

Proof related features

1. Automatic proof using an embedded ATP
2. Automatic proof using Coq
3. Interactive proof using Coq

Current situation

- GeoProof $\longrightarrow$ Coq
- Geoview $\longleftarrow$ Coq
- CoqWeb ? Geogebra

Perspectives

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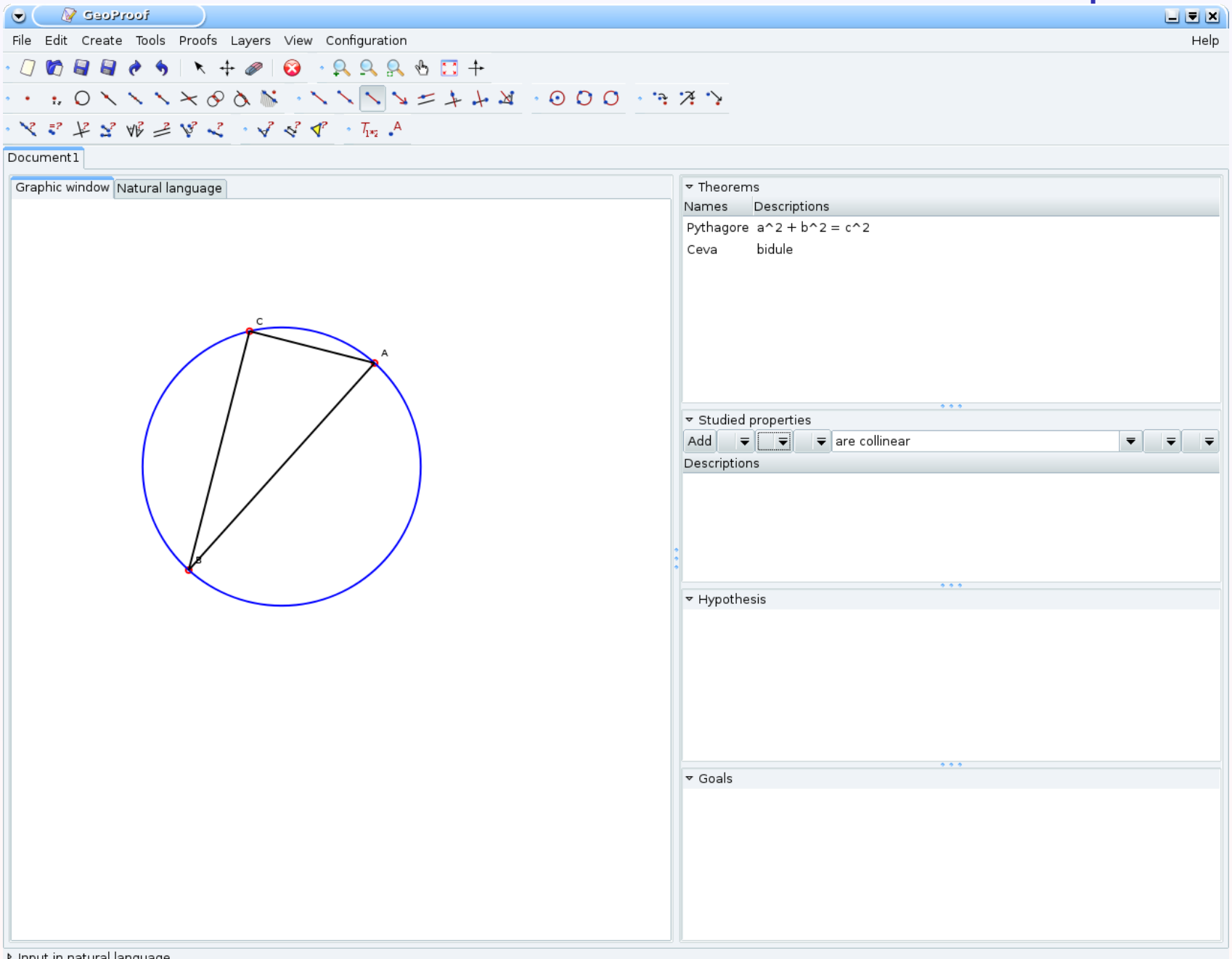
   - ...

A mock-up


GeoProof
File Edit Create Tools Proofs Layers View Configuration Help
Document1
Graphic window
Natural language
C
A
B
Theorems
Names
Descriptions
Pythagore
a^2 + b^2 = c^2
Ceva
bidule
Studied properties
Add
are collinear
Descriptions
Hypothesis
Goals
Input in natural language

What needs to be done (technically)

- Allow several views for the same document
- Implement macros
- Allow teacher to change the list of theorems
- Implement drag and drop
  - Conjecture $\longrightarrow$ Hypothesis : `assert`
  - Theorem $\longrightarrow$ Figure : unify and `apply`
  - Hypothesis $\longrightarrow$ Theorem : assert composition
  - Hypothesis $\longrightarrow$ Goal : `assumption`
  - New object on figure : apply theorem corresponding to the existence of the object, generates a sub goal.
  - Create a new macro : create a lemma corresponding to the existence theorem.
  - Use macro : `apply`
- Use Proof General 2 protocol

Some questions

- Impose forward style reasoning ? (like in Geometrix : `assert (th hyp1 hypn)`)
- How to reason modulo to hide complexity ?
- Always substitute when an equality is known ?
- How to display negative facts $((AB) \nparallel (CD))$?
- ...

Constructions

- A macro corresponds to the proof of a theorem stating the existence of a ruler and compass construction.
- Verify formally constructions generated by expert systems.

A repository for formal proofs in Geometry

- For visibility. . .
- Containing proofs of algorithms, theorems and meta-theorems.

- A name ?
- Which language ?
- Available on Galapagos' web site ?

The End.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